

Holographic Dual of de Sitter Universe with AdS Bubbles

Sugumi Kanno¹, Misao Sasaki^{2,4}, and Jiro Soda³

¹*Institute of Cosmology, Department of Physics and Astronomy,
Tufts University, Medford, Massachusetts 02155, USA*

²*Yukawa Institute, Kyoto University, Kyoto 606-8502, Japan*

³*Department of Physics, Kyoto University, Kyoto 606-8502, Japan*

⁴*Korea Institute for Advanced Study, Seoul 130-722, Republic of Korea*

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We study the proposal that a de Sitter (dS) universe with an Anti-de Sitter (AdS) bubble can be replaced by a dS universe with a boundary CFT. To explore this duality, we consider incident gravitons coming from the dS universe through the bubble wall into the AdS bubble in the original picture. In the dual picture, this process has to be identified with the absorption of gravitons by CFT matter. We have obtained a general formula for the absorption probability in general $d+1$ spacetime dimensions. The result shows the different behavior depending on whether spacetime dimensions are even or odd. We find that the absorption process of gravitons from the dS universe by CFT matter is controlled by localized gravitons (massive bound state modes in the Kaluza-Klein decomposition) in the dS universe. The absorption probability is determined by the effective degrees of freedom of the CFT matter and the effective gravitational coupling constant which encodes information of localized gravitons. We speculate that the dual of $(d+1)$ -dimensional dS universe with an AdS bubble is also dual to a d -dimensional dS universe with CFT matter.

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I. INTRODUCTION

The resolution of the spacetime singularity has been a long standing problem [1]. It is believed that quantum gravity will resolve the singularity, and string theory seems the most promising candidate for quantum gravity. So far, one of the most important outcomes from string theory is the AdS/CFT correspondence [2–4], or more generally the Gauge/Gravity correspondence.

There are two ways to use the AdS/CFT correspondence. The first one is to use the correspondence to understand the strongly coupled field theory using classical gravity. Remarkably, the correspondence has been used in the condensed matter context for calculating correlation functions in a strongly interacting field theory using the dual classical gravity description. The AdS/CFT correspondence can also be used the other way around. Namely, the AdS/CFT correspondence can be used to describe quantum gravity using a well-defined dual field theory. Since CFT is a healthy, unitary theory, it is natural to expect that the AdS/CFT correspondence resolves the singularity problem. Indeed, this view has been applied to the singularity problem in the past [5–8].

Even in the classical context, the singularity is annoying. Recently, it is found that black hole singularities can be treated by an excision method in numerical relativity [9]. It is interesting to see if this idea can be promoted to a theoretical framework. In fact, the singularity excision has been discussed in the context of black hole evaporation [10, 11]. In principle, the excision method can be formulated as an effective field theory with boundary, which can be improved with a renormalization group idea.

Interestingly, the above apparently two different approaches merged into a recent proposal by Maldacena that an AdS bubble in a dS spacetime can be replaced by a boundary CFT according to the AdS/CFT duality [12]. Since AdS spacetime is unstable to the formation of a singularity under a small perturbation, if this proposal is valid, we can avoid dealing directly with a singular spacetime by replacing it with the spacetime with boundary. In a sense, we have a stringy realization of the singularity excision.

Apparently, it is worth exploring this proposal in detail. A simple, direct route to the understanding of the proposal is to study incident gravitons from the outside dS universe into the AdS bubble. In the original picture, gravitons can penetrate into the AdS bubble. One can calculate the transmission probability for this process. How should we understand this process in the dual picture in which the AdS bubble is replaced by the boundary bubble wall with the dual CFT? Since there is nothing inside of the bubble wall, it is expected intuitively that the energy of gravitons would be transferred to particles of the CFT.

In fact in the limit of infinitely large dS radius (i.e., in the Minkowski limit), by calculating the transmission probability of gravitons coming from the dS side through the bubble wall, it is argued that the result can be interpreted as the absorption probability of gravitons into CFT particles [13]. However, the method used in [13] was specific to four dimensions and the detailed discussion was provided only for the case of an AdS bubble in Minkowski spacetime.

Here we are interested in more general situations. If Maldacena's conjecture is correct, it should hold in arbitrary

dimensions. We study the singularity excision in general $d + 1$ spacetime dimensions. In particular, we give much attention to the role of the dS universe in this dual picture. We note that, in the string landscape or in the multiverse picture [14–16], AdS bubbles will be formed in various kinds of dS vacua in (perhaps) various spacetime dimensions. Our analysis will be relevant to these cosmologically interesting situations.

The organization of this paper is as follows. In section II, we provide a model for AdS bubbles nucleating in the inflating multiverse and explain the dual picture where the AdS bubble is replaced with the boundary CFT. In section III, we calculate the transmission probability in arbitrary dimensions. In section IV, we identified the transmission probability with the absorption probability in the dual picture. In the decoupling limit of gravity in the sense of AdS/CFT correspondence, it turns out that the absorption probability is factorized into a factor from effective degrees of CFT matter, an effective gravitational coupling constant, and a kinematical factor. We obtain an explicit formula for various situations. We find the absorption probability is controlled by graviton bound states originating in de Sitter spacetime. The final section is devoted to the conclusion. In Appendix A, we summarize several useful mathematical formulas. In Appendix B, we give an exact formula for the transmission probability in four dimensions without taking the decoupling limit, in order to make precise comparison with the result obtained in [13]. In Appendix C, a computation of the bound states in the presence of an AdS bubble is given. In Appendix D, we discuss the wall fluctuation mode and compare it with bound states.

II. HOLOGRAPHIC EXCISION OF ADS BUBBLES

In the landscape picture, AdS bubbles nucleate with a certain probability in the inflating multiverse [17]. In this section, we pick up a part of the multiverse where the universe is in a dS vacuum. First, we describe an AdS bubble in the dS universe. Then, we present a dual of this geometry by following Maldacena [12].

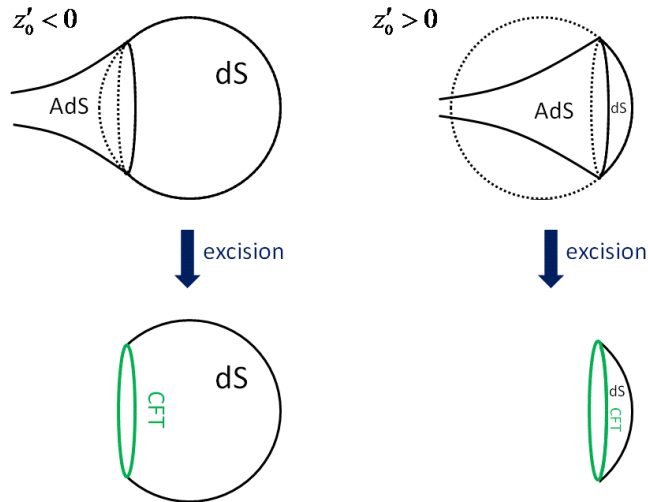


FIG. 1: The excision process is depicted. In the case $z'_0 < 0$, the AdS bubble does not hide the great circle of dS. While, in the case $z'_0 > 0$, there is no great circle in dS spacetime. The former describes an AdS bubble in the dS universe. The latter describes the case when both AdS and dS spacetimes are surrounded by a common boundary wall.

We begin with the background spacetime with a $(d + 1)$ -dimensional AdS bubble of curvature radius ℓ_{AdS} in a dS spacetime of curvature radius ℓ_{dS} . The metric we consider is

$$ds^2 = a^2(z) [dz^2 + ds_{\text{dS}}^2] , \quad (2.1)$$

where ds_{dS}^2 is the d -dimensional dS spacetime with unit curvature radius. Inside the bubble, the conformal factor is given by the AdS spacetime,

$$a(z) = \frac{\ell_{\text{AdS}}}{\sinh(z_0 - z)} , \quad z \leq 0 . \quad (2.2)$$

This region is separated by a domain wall from an outer region of the dS universe. Then outside the bubble is expressed by the dS spacetime,

$$a(z) = \frac{\ell_{\text{dS}}}{\cosh(z + z'_0)}, \quad z \geq 0. \quad (2.3)$$

Here, z_0 and z'_0 are the parameters that determine the AdS and dS curvature radii and the position of the bubble wall in each spacetime. We match these two spacetimes at $z = 0$. Note that $z_0 > 0$ while z'_0 can be either positive or negative. In Fig. 1, we explain the geometrical meaning of the parameter z'_0 . The continuity of these metrics leads to the relation,

$$\frac{\ell_{\text{AdS}}}{\sinh z_0} = \frac{\ell_{\text{dS}}}{\cosh z'_0} = a_w, \quad (2.4)$$

where a_w is the curvature radius of the domain wall which is a d -dimensional dS spacetime. The continuity condition Eq. (2.4) gives the relations $\sinh z_0 = \ell_{\text{AdS}}/a_w$ and $\cosh z'_0 = \ell_{\text{dS}}/a_w$. Thus, varying z_0 and z'_0 is equivalent to varying ℓ_{AdS} and ℓ_{dS} under fixed a_w . The junction condition at the domain wall gives the tension of the domain wall of the form,

$$T = T_{\text{cr}} \frac{\ell_{\text{AdS}}}{a_w} \left[\sqrt{1 + \left(\frac{a_w}{\ell_{\text{AdS}}} \right)^2} \pm \sqrt{1 - \left(\frac{a_w}{\ell_{\text{dS}}} \right)^2} \right], \quad T_{\text{cr}} = \frac{d-1}{\kappa^2 \ell_{\text{AdS}}}, \quad (2.5)$$

where the plus and minus signs in front of the second term correspond to the choice $z'_0 > 0$ and $z'_0 < 0$, respectively. We have defined $\kappa^2 = 8\pi G_{d+1}$ and T_{cr} is the tension corresponding to a flat domain wall $a_w \rightarrow \infty$. We note that the choice $z'_0 < 0$ is more natural in the sense that it corresponds to the Coleman-De Luccia bubble [17], that is, an AdS bubble nucleated in the dS universe. On the other hand, the choice $z'_0 > 0$ corresponds to the case when both AdS and dS spacetimes are surrounded by a common boundary wall. Although the latter may not be easily realizable if considered in the context of field theory, mathematically there is nothing wrong with such a configuration. Moreover, if we consider multiple nucleation processes, a dS bubble could be produced as a remnant of the false vacuum [18]. Therefore we consider both of these two cases in the following.

By following Maldacena's conjecture [12], we excise the AdS bubble and replace it with a boundary CFT. The resultant spacetime is dS bounded by a boundary dS on which CFT lives. There exists no spacetime beyond the bubble wall (see Fig. 1). In this paper, we study consequences of this conjectured duality through probe gravitons.

In order to distinguish the outer dS universe from the boundary dS where CFT resides, we refer the outer dS universe as dS bulk in the following.

III. TRANSMISSION PROBABILITY IN ARBITRARY DIMENSIONS

In this section, we calculate the transmission probability of incident gravitons from the dS bulk into the AdS bubble in arbitrary dimensions.

We consider gravitational waves on the background explained in the previous section. The conformal factor in this background is expressed by using the theta function as

$$a(z) = \theta(-z) \frac{\ell_{\text{AdS}}}{\sinh(z_0 - z)} + \theta(z) \frac{\ell_{\text{dS}}}{\cosh(z + z'_0)}. \quad (3.1)$$

The d -dimensional spacetime components (i, j) of the tensor perturbation, ψ_{ij} , are defined by

$$ds^2 = a^2(z) [dz^2 + ds_{\text{dS}}^2 + \psi_{ij} dx^i dx^j]. \quad (3.2)$$

The wave equation for ψ_{ij} is given by

$$\left[\frac{d^2}{dz^2} + \frac{d-1}{a} \frac{da}{dz} \frac{d}{dz} + (\square - 2) \right] \psi_{ij} = 0. \quad (3.3)$$

Here, \square is the d -dimensional d'Alembertian operator. After separation of variables by $\psi_{ij}(z, x^i) = \tilde{\psi}(z) Y_{ij}(x^i)$, we find Y_{ij} satisfies

$$[\square - 2 - M^2] Y_{ij} = 0, \quad \nabla^j Y_{ij} = Y^i_i = 0, \quad (3.4)$$

where M is a constant of separation which represents the Kaluza-Klein (KK) mass of the gravitons in d dimensions. Changing the variable: $\tilde{\psi} = \phi/a^{(d-1)/2}$, we obtain a Schrödinger type wave equation,

$$\left[-\frac{d^2}{dz^2} + V(z) \right] \phi = M^2 \phi, \quad (3.5)$$

where the effective potential V reads

$$\begin{aligned} V(z) &= \frac{1}{a^{(d-1)/2}} \frac{d^2 a^{(d-1)/2}}{dz^2} = \frac{d-1}{2a} \frac{d^2 a}{dz^2} + \frac{(d-1)(d-3)}{4a^2} \left(\frac{da}{dz} \right)^2 \\ &= -\frac{d-1}{2} \left[\coth z_0 + \tanh z_0' \right] \delta(z) \\ &\quad + \theta(-z) \left[\frac{(d-1)^2}{4} + \frac{d^2-1}{4 \sinh^2(z-z_0)} \right] + \theta(z) \left[\frac{(d-1)^2}{4} - \frac{d^2-1}{4 \cosh^2(z+z_0')} \right]. \end{aligned} \quad (3.6)$$

Note that the first term proportional to the delta function corresponds to the tension of the domain wall, Eq. (2.5).

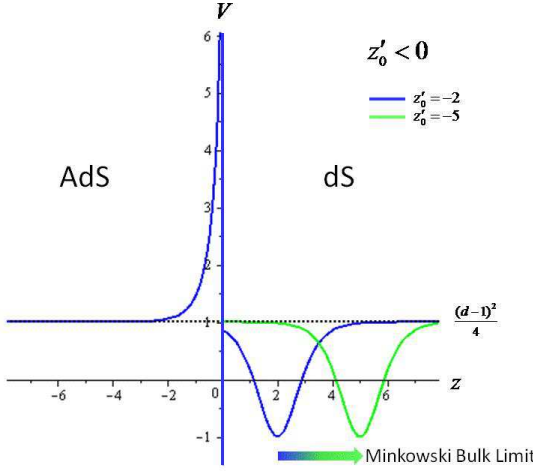


FIG. 2: The potential for $z'_0 < 0$.

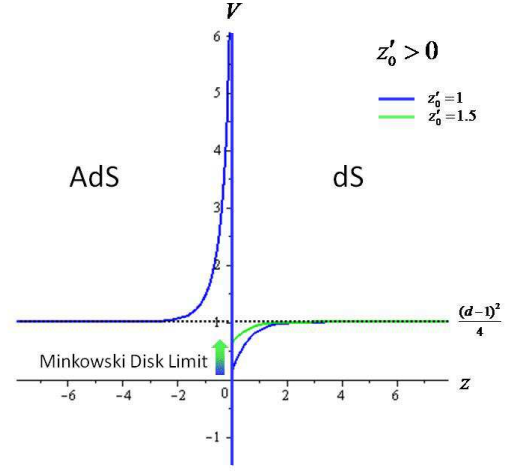


FIG. 3: The potential for $z'_0 > 0$.

In Figs. 2 and 3, we depict the above potential for $z'_0 < 0$ and $z'_0 > 0$, respectively. At the position of the domain wall $z = 0$, there is an infinitely deep well due to a minus of the delta function. We can see that, for $z'_0 < 0$, the potential well in the dS bulk is moving away from the position of the domain wall as the dS radius gets larger and eventually approaches the Minkowski bulk. On the other hand for $z'_0 > 0$, the potential well at the domain wall becomes shallow as the dS radius gets larger and eventually approaches a Minkowski disk.

Now we want to consider incident gravitational waves from the dS bulk. The boundary condition we consider is

$$\phi(z) \rightarrow \mathbf{T} e^{-ikz}; \quad z \rightarrow -\infty, \quad (3.7)$$

$$\phi(z) \rightarrow \mathbf{R} e^{ikz} + e^{-ikz}; \quad z \rightarrow \infty, \quad (3.8)$$

where we have defined

$$k \equiv \sqrt{M^2 - \frac{(d-1)^2}{4}} \quad \leftrightarrow \quad M^2 = k^2 + \frac{(d-1)^2}{4}, \quad (3.9)$$

and \mathbf{T} and \mathbf{R} represent the amplitudes of transmission and reflection waves, respectively.

A. Solutions inside AdS bubble

It is easy to see that the general solution is given in terms of the associated Legendre functions inside AdS bubble. By imposing the boundary condition (3.7), we find the solution is given by

$$\phi(z) = c_1 P_\nu^{ik}(\coth(z_0 - z)) ; \quad c_1 \equiv \mathbf{T} \Gamma(1 - ik) e^{-ikz_0}, \quad \nu \equiv \frac{d-1}{2}. \quad (3.10)$$

Note that we consider the case $M^2 > (d-1)^2/4$, thus we take an imaginary number ik as the order of the associated Legendre function. Since there are several different normalizations of $P_\nu^\mu(u)$, in order to avoid confusion, we spell out the definitions adopted in this paper and summarize other useful formulas related to the associated Legendre functions in Appendix A.

B. Solutions in de Sitter bulk

In the dS bulk, the general solution is also given by the associated Legendre functions. After imposing the boundary condition, (3.8), we obtain the solution of the form,

$$\phi(z) = d_1 P_\nu^{ik}(\tanh(z + z'_0)) + d_2 P_\nu^{-ik}(\tanh(z + z'_0)), \quad (3.11)$$

where $\nu = (d-1)/2$ as before, and

$$d_1 \equiv \mathbf{R} \Gamma(1 - ik) e^{\pi k} e^{-ikz'_0}, \quad d_2 \equiv \Gamma(1 + ik) e^{-\pi k} e^{ikz'_0}. \quad (3.12)$$

C. Transmission probability

In order to solve the scattering problem, we first match the solutions obtained in Eqs.(3.10) and (3.11) smoothly at the domain wall. This requires the following two junction conditions at $z = 0$:

$$c_1 P_\nu^{ik}(x_0) = d_1 P_\nu^{ik}(x'_0) + d_2 P_\nu^{-ik}(x'_0), \quad (3.13)$$

$$\begin{aligned} c_1 [(\nu+1)x_0 P_\nu^{ik}(x_0) - (\nu+1-ik)P_{\nu+1}^{ik}(x_0) + \nu(x_0+x'_0)P_\nu^{ik}(x_0)] \\ = -d_1 [(\nu+1)x'_0 P_\nu^{ik}(x'_0) - (\nu+1-ik)P_{\nu+1}^{ik}(x'_0)] - d_2 [(\nu+1)x'_0 P_\nu^{-ik}(x'_0) - (\nu+1+ik)P_{\nu+1}^{-ik}(x'_0)] \end{aligned} \quad (3.14)$$

where we have defined

$$x_0 \equiv \coth z_0, \quad x'_0 \equiv \tanh z'_0. \quad (3.15)$$

We note that the ranges of these variables are $|x_0| > 1$ and $|x'_0| < 1$ by definition.

From the asymptotic forms of Eqs. (3.7) and (3.8), we see the transmission probability is given by $|\mathbf{T}|^2$. By solving above continuous conditions, we find the transmission probability

$$\mathcal{T} \equiv |\mathbf{T}|^2 = \left| \frac{c_1}{\Gamma(1-ik)} \right|^2 = \frac{4}{\pi^2} \sinh^2 \pi k e^{-2\pi k} |\mathcal{M}|^2, \quad (3.16)$$

where

$$\begin{aligned} \mathcal{M}^{-1}(x_0, x'_0, k) &= P_\nu^{ik}(x'_0) [(\nu+1-ik)P_{\nu+1}^{ik}(x_0) - (\nu+1)x_0 P_\nu^{ik}(x_0) - \nu(x_0+x'_0)P_\nu^{ik}(x_0)] \\ &\quad + P_\nu^{ik}(x_0) [(\nu+1-ik)P_{\nu+1}^{ik}(x'_0) - (\nu+1)x'_0 P_\nu^{ik}(x'_0)] \\ &= -(\nu+ik)(P_\nu^{ik}(x_0)P_{\nu-1}^{ik}(x'_0) + P_{\nu-1}^{ik}(x_0)P_\nu^{ik}(x'_0)), \end{aligned} \quad (3.17)$$

where we have used a recurrence relation: $(\nu+1-ik)P_{\nu+1}^{ik}(x) - (2\nu+1)xP_\nu^{ik}(x) + (\nu+\mu)P_{\nu-1}^{ik}(x) = 0$ at the last step. It should be stressed that this is quite a general result.

IV. DUAL INTERPRETATION

There are three relevant length scales in the system, namely, ℓ_{dS} , ℓ_{AdS} , and a_w . Since we know $\ell_{\text{dS}} = a_w \cosh z'_0$ from the junction condition (2.4), we see ℓ_{dS} is always larger than a_w . So far we have imposed no restriction on the model parameters.

We are interested in the regime where physical momenta of gravitons are much smaller than the AdS scale, $k/a_w \ll 1/\ell_{\text{AdS}}$. This is guaranteed for a finite value of k if we consider the case of the large bubble limit,

$$\frac{a_w}{\ell_{\text{AdS}}} = \frac{1}{\sinh z_0} \gg 1, \quad (4.1)$$

which is equivalent to $z_0 \ll 1$ or $x_0 \gg 1$ according to the definition of x_0 in Eq. (3.15). This means the case when the dS boundary is located almost at infinity in the AdS spacetime and hence the decoupling limit of gravity from the boundary CFT [19, 20]. There, the AdS/CFT correspondence is supposed to be exact and we may consider the boundary CFT instead of the gravity in the AdS bubble. This is why we can replace the AdS bubble by the boundary CFT. Then we can move on to the dual picture of no AdS spacetime inside the bubble and identify the absorption probability of CFT particles $\mathcal{A}(k)$ with the transmission probability of gravitons from the dS bulk $\mathcal{T}(k)$,

$$\mathcal{A}(k) = \mathcal{T}(k). \quad (4.2)$$

Under the above assumption, Eq. (3.17) is simplified as

$$\mathcal{M}^{-1}(x_0, x'_0, k) = -(\nu + ik) P_\nu^{ik}(x_0) P_{\nu-1}^{ik}(x'_0). \quad (4.3)$$

We notice that the dependences of the AdS bubble on x_0 and the dS bulk on x'_0 are factorized in this case. Using the asymptotic expansion of $P_\nu^{ik}(x_0)$ at $x_0 \gg 1$, we obtain

$$\begin{aligned} |\mathcal{M}|^{-2} &= (\nu^2 + k^2) \frac{\sinh^2 \pi k}{\pi^2 k^2} e^{-2\pi k} |F_{\nu,k}(x_0)|^2 |F_{\nu-1,k}(x'_0)|^2 \\ &\stackrel{x_0 \gg 1}{=} \left(\frac{a_w}{\ell_{\text{AdS}}} \right)^{2\nu} 2^{2\nu} \frac{\sinh^2 \pi k}{\pi^3} e^{-2\pi k} \frac{\Gamma(\nu + \frac{1}{2})}{\Gamma(\nu)} \left| \frac{\Gamma(\nu)\Gamma(-ik)}{\Gamma(\nu - ik)} \right|^2 |F_{\nu-1,k}(x'_0)|^2, \end{aligned} \quad (4.4)$$

where for notational simplicity we have introduced a function $F_{\nu,k}(x)$ by

$$F_{\nu,k}(x) \equiv F\left(-\nu, \nu + 1, 1 - ik; \frac{1 - x}{2}\right), \quad (4.5)$$

where $F(\alpha, \beta, \gamma, u)$ is a hypergeometric function. Plugging the above into Eq. (3.16), the absorption probability is expressed as

$$\begin{aligned} \mathcal{A}(k) &= 2^{2-2\nu} \pi \left(\frac{\ell_{\text{AdS}}}{a_w} \right)^{2\nu} \frac{\Gamma^2(\nu)}{\Gamma^2(\nu + \frac{1}{2})} \left| \frac{\Gamma(\nu - ik)}{\Gamma(\nu)\Gamma(-ik)} \right|^2 |F_{\nu-1,k}(x'_0)|^{-2} \\ &= 2^{2-2\nu} k \sinh \pi k \left(\frac{\ell_{\text{AdS}}}{a_w} \right)^{2\nu} \frac{\Gamma^2(\nu)}{\Gamma^2(\nu + \frac{1}{2})} \left[\prod_{n=0}^{\infty} \left(1 + \frac{k^2}{(\nu + n)^2} \right) \right]^{-1} |F_{\nu-1,k}(x'_0)|^{-2}. \end{aligned} \quad (4.6)$$

Let us consider the case of four dimensions ($d = 3$ or $\nu = 1$). Since the factor $F_{0,k}(x'_0) = 1$ is independent of x'_0 , using the first line of Eq. (4.6), the absorption probability is easily found to be

$$\mathcal{A} = 4k^2 \left(\frac{\ell_{\text{AdS}}}{a_w} \right)^2. \quad (4.7)$$

The energy of KK gravitons on the wall is given by Eq. (3.9) with $d = 3$,

$$E^2 = \frac{M^2}{a_w^2} = k_w^2 + \lambda_{\text{IR}}^2, \quad (\Delta E)^2 = E^2 - \lambda_{\text{IR}}^2 = k_w^2, \quad (4.8)$$

where we have defined $k_w = k/a_w$ and $\lambda_{\text{IR}} = 1/a_w$. Then Eq. (4.7) reads

$$\mathcal{A} = c_4 G_4 (\Delta E)^2, \quad c_4 = 4 \frac{\ell_{\text{AdS}}^2}{G_4}, \quad (4.9)$$

where we have introduced the four-dimensional gravitational coupling G_4 and degrees of freedom c_4 of the CFT according to the AdS/CFT correspondence [2–4]. The effect of the AdS spacetime comes in only through this c_4 . The factor $G_4(\Delta E)^2$ is the decay rate of gravitons into a single degree of freedom of the CFT. This result tells us that gravitons coming from the four-dimensional dS bulk decay into CFT particles of three dimensions. This is a standard decay rate in the Minkowski spacetime, which is in agreement with the low energy limit of the result obtained in [13].¹

It is known that bound states appear in the transmission amplitude as poles. In fact, we find the poles in the factor from the dS bulk $P_{\nu-1}^{ik}(x'_0)$ in the complex k -plane in Eq. (4.6) from the calculation of the bound state, Eq. (C8), in Appendix C. Also curiously, there appear other poles in the factor of the infinite product coming from the AdS bubble which are not related to the bound states, depending on whether spacetime dimensions are even or odd. Let us see this next.

In the case of even spacetime dimensions (or odd d) equal to or more than four, $d = 2m + 3$ or $\nu = m + 1$ ($m = 0, 1, 2, \dots$), Eq. (4.6) reduces to

$$\mathcal{A}(k) = 4k^2 \left(\frac{\ell_{\text{AdS}}}{a_w} \right)^{2m+2} \left[\frac{m!}{(2m+1)!!} \right]^2 \prod_{n=1}^m \left(1 + \frac{k^2}{n^2} \right) \left| F_{m,k}(x'_0) \right|^{-2}, \quad (4.10)$$

where we have used the second line of Eq. (4.6) and rewritten the infinite product in it as

$$\left[\prod_{n=1}^{\infty} \left(1 + \frac{k^2}{(m+n)^2} \right) \right]^{-1} = \prod_{n=1}^m \left(1 + \frac{k^2}{n^2} \right) \frac{\pi k}{\sinh \pi k}. \quad (4.11)$$

We see that written in this way the infinite product looks like a thermal distribution function obeying Bose statistics. Going back to the absorption probability given by the second line of Eq. (4.6), the factor $\sinh \pi k$ there is canceled by the above $1/\sinh \pi k$ factor, leaving only a finite number of zeros at $ik = 1, 2, \dots, m$. We speculate that the original factor $\sinh \pi k$ in the second line of Eq. (4.6) is something to do with the nature of the wall, while the $1/\sinh \pi k$ factor in Eq. (4.11) indeed represents the thermal spectrum due to the acceleration of the boundary wall: Since the boundary wall is a dS spacetime with dS temperature $T_{\text{dS}} = (2\pi a_w)^{-1}$, which can be also regarded as the Unruh temperature due to its acceleration relative to the bulk, observers on the boundary wall see a thermal spectrum of gravitons. This thermal spectrum is known to be in the form of Bose statistics in even spacetime dimensions, i.e., for odd d , while in the form of Fermi statistics in odd spacetime dimensions, i.e., for even d [21]. The above argument is in fact supported if we consider the case of odd spacetime dimensions, which we now turn to.

In the case of odd spacetime dimensions (or even d) equal to or more than five, $d = 2m + 4$ or $\nu = m + 3/2$ ($m = 0, 1, 2, \dots$), Eq. (4.6) gives

$$\mathcal{A}(k) = \frac{\pi k \tanh \pi k}{2^{4m+3}} \left(\frac{\ell_{\text{AdS}}}{a_w} \right)^{2m+3} \left[\frac{(2m+1)!!}{(m+1)!} \right]^2 \prod_{n=0}^m \left(1 + \frac{4k^2}{(2n+1)^2} \right) \left| F_{m+1/2,k}(x'_0) \right|^{-2}, \quad (4.12)$$

where we have used

$$\left[\prod_{n=1}^{\infty} \left(1 + \frac{4k^2}{(2m+1+2n)^2} \right) \right]^{-1} = \prod_{n=0}^m \left(1 + \frac{4k^2}{(2n+1)^2} \right) \frac{1}{\cosh \pi k}. \quad (4.13)$$

We find that zeros due to the factor $\sinh \pi k$ in Eq. (4.6) remain and consequently poles in the form $\tanh \pi k$ due to the factor $1/\cosh \pi k$ in the infinite product in Eq. (4.13) appear. Thus, an infinite number of poles which are not related to graviton bound states show up in odd spacetime dimensions. The factor $1/\cosh \pi k$ represents a thermal spectrum of Fermi statistics. This difference from even spacetime dimensions can be attributed to the absence of Huygens principle in odd spacetime dimensions. As mentioned in the case of even spacetime dimensions, the above analysis is consistent with the fact that an accelerated observer see a thermal spectrum at Unruh temperature, with the spectrum obeying Bose (Fermi) statistics in even (odd) spacetime dimensions.

Now, we try to express the absorption probability in an analogous form to Eq. (4.9).

¹ Note that the result (4.7) or (4.9) is unbounded as energy increases, apparently violating the unitarity bound. However, this is simply due to our assumption that $k/a_w \ll 1/\ell_{\text{AdS}}$. If we relax this assumption, the result is perfectly finite in the limit of the infinitely large energy. For completeness, we derive an exact expression for the transmission probability in four dimensions without any approximation in Appendix B. It agrees with [13] in the limit of the Minkowski bulk.

In the case of even spacetime dimensions (or odd d) equal to or more than four, $d = 2m + 3$ ($m = 0, 1, 2, \dots$), Eq. (4.10) may be expressed in the form,

$$\mathcal{A}(k) = c_{2m+4} \mathcal{G}_{2m+4} k_w^2 \prod_{n=1}^m (n^2 \lambda_{\text{IR}}^2 + k_w^2) . \quad (4.14)$$

Here, we have defined the $(2m + 4)$ -dimensional degree of freedom c_{2m+4} of the CFT, and the $(2m + 4)$ -dimensional effective gravitational coupling constant, \mathcal{G}_{2m+4} , which are expressed as

$$\begin{aligned} c_{2m+4} &= \left[\frac{2}{(2m+1)!!} \right]^2 \frac{\ell_{\text{AdS}}^{2m+2}}{G_{2m+4}} , \\ \mathcal{G}_{2m+4} &= G_{2m+4} \left| F_{m,k}(x'_0) \right|^{-2} . \end{aligned} \quad (4.15)$$

We note that c_{2m+4} stems from the AdS/CFT correspondence and \mathcal{G}_{2m+4} includes the information of the dS bulk geometry, which is different from the gravitational coupling constant in the dS bulk G_{2m+4} . As we will see later in subsection IV D, the difference between \mathcal{G}_{2m+4} and G_{2m+4} may be attributed to the effect of localization of gravitons in the dS bulk.

In the case of odd spacetime dimensions (or even d) equal to or more than five, $d = 2m + 4$ ($m = 0, 1, 2, \dots$), Eq. (4.12) gives

$$\mathcal{A}(k) = c_{2m+5} \mathcal{G}_{2m+5} k_w \tanh \pi k \prod_{n=0}^m \left(\frac{(2n+1)^2}{4} \lambda_{\text{IR}}^2 + k_w^2 \right) , \quad (4.16)$$

where we have defined

$$\begin{aligned} c_{2m+5} &= \frac{\pi}{2^{2m+1}} \left[\frac{1}{(m+1)!} \right]^2 \frac{\ell_{\text{AdS}}^{2m+3}}{G_{2m+5}} , \\ \mathcal{G}_{2m+5} &= G_{2m+5} \left| F_{m+1/2,k}(x'_0) \right|^{-2} . \end{aligned} \quad (4.17)$$

As we noted earlier below Eq. (4.3), the factorized dependence on the dS bulk (x'_0 or ℓ_{dS}) and the AdS spacetime (x_0 or ℓ_{AdS}) is encoded in \mathcal{G}_{d+1} and c_{d+1} in the CFT language, respectively. In fact, if we consider the AdS bubble in the Minkowski spacetime, we find there is no correction to the effective gravitational constant, $\mathcal{G}_{d+1} = G_{d+1}$.² This is because we took the decoupling limit.

We now look at the absorption probability in three special cases in which the above analytic formula can be simplified:

- Minkowski disc in AdS spacetime:
 $\ell_{\text{dS}}/a_w = \cosh z'_0 \rightarrow \infty$, or $x'_0 = \tanh z'_0 = 1$.
- AdS bubble in the Minkowski spacetime:
 $\ell_{\text{dS}}/a_w = \cosh z'_0 \rightarrow \infty$, or $x'_0 = \tanh z'_0 = -1$.
- The largest possible bubbles in the dS bulk:
 $\ell_{\text{dS}} = a_w \cosh z'_0 = a_w$, or $x'_0 = 0$.

More generic cases can be investigated perturbatively and numerically. We discuss the above three cases separately.

A. Minkowski disc in AdS spacetime

We consider the case of a Minkowski disc surrounded by the domain wall with radius a_w . That is, $\ell_{\text{dS}}/a_w \rightarrow \infty$ and $z'_0 \rightarrow +\infty$. From Eqs. (2.4) and (3.15), this corresponds to

$$x'_0 = \tanh z'_0 = 1 . \quad (4.18)$$

² As we will see below in Section IV B, however, if we first consider a dS bulk and then take the Minkowski bulk limit, this statement does not hold in the case of odd spacetime dimensions (even d). Regrettably we do not have a convincing explanation for this discrepancy.

In this case, the effective gravitational coupling \mathcal{G}_{d+1} becomes G_{d+1} because of $F(a, b, c; 0) = 1$, where a, b and c are arbitrary real or complex parameters.

In the case of even spacetime dimensions, $d = 2m + 3$, we find

$$\mathcal{A}(k) = c_{2m+4} G_{2m+4} k_w^2 \prod_{n=1}^m (n^2 \lambda_{\text{IR}}^2 + k_w^2) . \quad (4.19)$$

We can easily check that Eq. (4.9) can be reproduced from this result in the case of four dimensions ($d = 3$, or $m = 0$). We find corrections caused by the curvature radius of the domain wall $\lambda_{\text{IR}}^2 = 1/a_w^2$ come into the result. However, taking the flat wall limit: $a_w \cdot k_w \rightarrow \infty$ with k_w fixed, we obtain the standard result in the Minkowski spacetime. In the case of six dimensions ($d = 5$, or $m = 1$), Eq. (??) becomes

$$\begin{aligned} \mathcal{A}(k) &= c_6 G_6 k_w^2 (\lambda_{\text{IR}}^2 + k_w^2) \\ &\xrightarrow{\text{flat wall}} c_6 G_6 (\Delta E)^4 . \end{aligned} \quad (4.20)$$

In the case of odd spacetime dimensions, $d = 2m + 4$, Eq. (4.16) gives

$$\mathcal{A}(k) = c_{2m+5} G_{2m+5} k_w \tanh \pi k \prod_{n=0}^m \left(\frac{(2n+1)^2}{4} \lambda_{\text{IR}}^2 + k_w^2 \right) . \quad (4.21)$$

We see that there are corrections due to the curvature radius of the domain wall λ_{IR}^2 again and also the thermal nature of the wall $\tanh \pi k$ appears in the result. However, if we take the flat wall limit, then $\tanh \pi k \rightarrow 1$. So all the corrections disappear in this limit.

In particular in the case of five dimensions ($d = 4$, or $m = 0$), we have

$$\begin{aligned} \mathcal{A}(k) &= c_5 G_5 k_w \tanh \pi k \left(\frac{1}{4} \lambda_{\text{IR}}^2 + k_w^2 \right) \\ &\xrightarrow{\text{flat wall}} c_5 G_5 (\Delta E)^3 . \end{aligned} \quad (4.22)$$

B. AdS bubble in Minkowski spacetime

We now consider an AdS bubble in Minkowski spacetime. That is, $\ell_{\text{dS}}/a_w \rightarrow \infty$ and $z'_0 \rightarrow -\infty$. From Eqs. (2.4) and (3.15), this corresponds to

$$x'_0 = \tanh z'_0 = -1 . \quad (4.23)$$

In fact, in the case of even spacetime dimensions, $d = 2m + 3$, we find the hypergeometric function in Eq. (4.15) is still unity, $|F_{m,k}(-1)| = |F(-m, m+1, 1-ik, 1-0)| = 1$. Hence we obtain exactly the same result as in the case of a Minkowski disc in AdS spacetime,

$$\mathcal{A}(k) = c_{2m+4} G_{2m+4} k_w^2 \prod_{n=1}^m (n^2 \lambda_{\text{IR}}^2 + k_w^2) . \quad (4.24)$$

In the case of four dimension ($d = 3$, or $m = 0$), we get the same result as Eq. (4.9). Also, we have Eq. (4.20) in six dimension ($d = 5$, or $m = 1$). Thus the standard result in the Minkowski spacetime is also applied to this case in the flat wall limit.

However, in the case of odd spacetime dimensions, $d = 2m + 4$, the situation is a bit more complicated. The value of the hypergeometric function in Eq. (4.16) in the limit $x'_0 \rightarrow -1$ takes the form,

$$F_{m+1/2,k}(x'_0) \xrightarrow{x'_0 \rightarrow -1} F\left(-m - \frac{1}{2}, m + \frac{3}{2}, 1-ik; 1-0\right) = \frac{e^{i\theta(k)}}{\sinh \pi k} \left(\cosh \pi k + e^{2ikz'_0} e^{i\varphi(k)} \right) , \quad (4.25)$$

where the phases $\theta(k)$ and $\varphi(k)$ depend on k in a complicated way. But the important point is the phase factor $e^{ikz'_0}$. It oscillates indefinitely in the limit $z'_0 \rightarrow -\infty$. This seems to imply the loss of rigorous information. Namely, one may not be able to recover the whole information of the bulk in this case. In any case, the result is

$$\left| F\left(-m - \frac{1}{2}, m + \frac{3}{2}, 1-ik; 1-0\right) \right|^{-2} = \frac{\sinh^2 \pi k}{\cosh^2 \pi k + 1} . \quad (4.26)$$

Thus, the effective gravitational coupling in Eq. (4.17) includes the effect of the dS bulk such as

$$\mathcal{G}_{2m+5} = G_{2m+5} \frac{\sinh^2 \pi k}{\cosh^2 \pi k + 1}. \quad (4.27)$$

It turns out that the \mathcal{G}_{2m+5} is smaller than G_{2m+5} .

The absorption probability is given by

$$\mathcal{A}(k) = c_{2m+5} \mathcal{G}_{2m+5} k_w \tanh \pi k \prod_{n=0}^m \left(\frac{(2n+1)^2}{4} \lambda_{\text{IR}}^2 + k_w^2 \right). \quad (4.28)$$

Compared to the case of the Minkowski disc, Eq. (4.21), we see that the absorption probability is smaller in the present case due to the suppressed effective gravitational constant (4.27). We found corrections coming from the curvature radius of the domain wall λ_{IR}^2 and also the thermal nature of the dS bulk $\tanh \pi k$ in the result. However, taking the flat wall limit: $a_w \cdot k_w \rightarrow \infty$ with k_w fixed, then $\tanh \pi k \rightarrow 1$, $\sinh^2 \pi k / (\cosh^2 \pi k + 1) \rightarrow 1$. Again, all the corrections disappear in this limit.

In particular in the case of five dimensions ($d = 4$, or $m = 0$), we obtain

$$\begin{aligned} \mathcal{A}(k) &= c_5 \mathcal{G}_5 k_w \tanh \pi k \left(\frac{1}{4} \lambda_{\text{IR}}^2 + k_w^2 \right) \\ &\xrightarrow{\text{flat wall}} c_5 G_5 (\Delta E)^3. \end{aligned} \quad (4.29)$$

C. Maximal radius bubbles in dS universe

Now, we consider the largest possible bubbles in the dS bulk. That is, $\ell_{\text{dS}} = a_w$ or $z'_0 = 0$. From Eq. (3.15), this corresponds to

$$x'_0 = \tanh z'_0 = 0. \quad (4.30)$$

In this limit, the hypergeometric function $F_{\nu-1,k}(x'_0)$ in the formulas (4.15) and (4.17) can be evaluated by using the formula,

$$F\left(-a, 1+a; b; \frac{1}{2}\right) = \frac{2^{1-b} \sqrt{\pi} \Gamma(b)}{\Gamma\left(\frac{b-a}{2}\right) \Gamma\left(\frac{b+a+1}{2}\right)}. \quad (4.31)$$

For even spacetime dimensions, $d = 2m + 3$, for which $a = m$, the above can be explicitly evaluated in terms of elementary functions. The result is

$$|F_{m,k}(0)|^{-2} = \left| F\left(-m, 1+m; 1-ik; \frac{1}{2}\right) \right|^{-2} = \begin{cases} \prod_{\ell=1}^p \frac{4\ell^2 + k^2}{(2\ell-1)^2 + k^2}; & m = 2p, \quad p = 0, 1, 2, \dots \\ \prod_{\ell=0}^p \frac{(2\ell+1)^2 + k^2}{4\ell^2 + k^2}; & m = 2p+1 \end{cases} \quad (4.32)$$

Thus, the effective gravitational coupling in even spacetime dimensions in Eq. (4.15) is enhanced when $m = 2p$ as

$$\mathcal{G}_{2m+4} = G_{2m+4} \prod_{\ell=1}^p \frac{4\ell^2 + k^2}{(2\ell-1)^2 + k^2}, \quad (4.33)$$

and when $m = 2p + 1$ as

$$\mathcal{G}_{2m+4} = G_{2m+4} \prod_{\ell=0}^p \frac{(2\ell+1)^2 + k^2}{4\ell^2 + k^2}. \quad (4.34)$$

We find that \mathcal{G}_{2m+4} is always greater than G_{2m+4} for both cases. Hence, the form of the absorption probability is the same as in the previous two Minkowski cases, but it is enhanced compared to them due to the enhancement in the

effective gravitational constant. We again find corrections caused by the curvature radius of the domain wall, which vanish in the flat wall limit $k = a_w k_w \rightarrow \infty$ with k_w fixed. Note that we have poles in Eqs. (4.33) and (4.34), which are related to the bound states discussed in Appendix C, Eq. (C8).

For four dimensions ($d = 3$ or $m = 0$), Eq. (4.9) is recovered from the above result because $F_{0,k}(x'_0) = 1$ independent of x'_0 as we mentioned earlier.

In the case of six dimensions ($d = 5$ or $m = 1$), the effect of the dS bulk is encoded in \mathcal{G}_6 in Eq. (4.34). So from Eq. (4.14) we obtain a slightly enhanced absorption probability compared to the previous two Minkowski cases,

$$\begin{aligned} \mathcal{A}(k) &= c_6 \mathcal{G}_6 k_w^2 (\lambda_{\text{IR}}^2 + k_w^2) = c_6 G_6 (\lambda_{\text{IR}}^2 + k_w^2)^2 \\ &\xrightarrow{\text{flat wall}} c_6 G_6 (\Delta E)^4. \end{aligned} \quad (4.35)$$

In the case of odd dimensions, $d = 2m + 4$, we put $a = m + 1/2$ in the formula (4.31). In this case there does not seem to exist a way to express the hypergeometric factor in terms of elementary functions. So we are unable to express the effective gravitational coupling \mathcal{G}_{2m+5} in a simple form. Nevertheless, we can estimate whether it is possible to describe the result in terms of the CFT language or not by considering the product of the absorption probability of two adjacent odd spacetime dimensions.

For definiteness, let us consider five dimensions ($d = 4$) and seven dimensions ($d = 6$). Here, as there are no gravitational waves in three dimensions ($d = 2$), we have skipped this case. If the result could be described by the CFT language, we expect the product of them should also be described by it. The result we obtain from Eq. (4.16) is

$$\begin{aligned} \mathcal{A}_{d=4} \times \mathcal{A}_{d=6} &= c_5 \mathcal{G}_5 c_7 \mathcal{G}_7 k_w^2 \tanh^2 \pi k \left(\frac{1}{4} \lambda_{\text{IR}}^2 + k_w^2 \right)^2 \left(\frac{9}{4} \lambda_{\text{IR}}^2 + k_w^2 \right) \\ &= c_5 G_5 c_7 G_7 \tanh^4 \pi k \left(\frac{1}{4} \lambda_{\text{IR}}^2 + k_w^2 \right)^2 \left(\frac{9}{4} \lambda_{\text{IR}}^2 + k_w^2 \right)^2 \\ &\xrightarrow{\text{flat wall}} c_5 G_5 (\Delta E)^3 \cdot c_7 G_7 (\Delta E)^5. \end{aligned} \quad (4.36)$$

Here again, we have corrections due to the curvature radius of the domain wall λ_{IR}^2 and also to the thermal nature of the domain wall. Note, however, there is an additional factor $\tanh^2 \pi k$ in the result. This may be due to the thermal nature of the dS bulk that was absent in the Minkowski bulk limit.

If we take the flat wall limit: $k = a_w k_w \rightarrow \infty$ with k_w fixed, all the corrections again disappear and the result is written by the product form of each absorption probability in five and seven dimensions. Thus we expect that the standard result in the Minkowski spacetime is recovered in the flat wall limit.

D. Generic cases

In four dimensions, we did not see any effect of corrections to the decay rate. This is probably related to the fact that three-dimensional gravity on the domain wall is trivial. In higher dimensions, we found that the dependence on a_w appears in the absorption probability explicitly, which vanishes in the flat wall limit: $k = a_w k_w \rightarrow \infty$ with k_w fixed, in all the cases studied in the previous subsections.

In the present model, we took the decoupling limit in the AdS side. Hence, the CFT would not couple to gravity on the domain wall if there were no dS bulk beyond the wall. Here, however, there is a dS bulk outside the AdS bubble and there are localized gravitons in the dS bulk (see Appendix C). Thus, we intuitively expect that not only corrections due to the curvature of the domain wall a_w but also effects of localized gravitons in the dS bulk with ℓ_{dS} would come in to the absorption probability. We studied analytically the cases of the Minkowski disc, the Minkowski bulk and the maximal radius bubble. In the former two cases there remains no effect of ℓ_{dS} because the limit $\ell_{\text{dS}} \rightarrow \infty$ is taken, and in the last case of the maximal radius bubble in the dS bulk, we cannot clearly see effects of ℓ_{dS} either because the bubble radius and the dS bulk curvature radius is degenerate. This is the reason why we could see only the corrections due to a_w in the previous subsections.

In order to reveal the effect of the dS bulk on the absorption probability we consider more generic cases in this subsection.

We first study the effect of dS bulk in the vicinity of the Minkowski disc limit. By expanding the hypergeometric function around $x'_0 = 1$, we see leading corrections due to finite curvature radius of the dS bulk to the effective gravitational coupling constant. In the case of even spacetime dimensions more than three, $d = 2m + 3$, we have

$$\mathcal{G}_{2m+4} = G_{2m+4} \left[1 + \frac{m(m+1)}{2(1+k^2)} \left(\frac{a_w}{\ell_{\text{dS}}} \right)^2 \right]. \quad (4.37)$$

We see that the effect is to enhance the gravitational constant in general, except for four dimensions. We find there is no correction in four dimensions ($m = 0$). In the case of odd spacetime dimensions more than four, $d = 2m + 4$, we have

$$\mathcal{G}_{2m+5} = G_{2m+5} \left[1 + \frac{(2m+1)(2m+3)}{4(1+k^2)} \left(\frac{a_w}{\ell_{\text{dS}}} \right)^2 \right]. \quad (4.38)$$

Also in this case we find the effect of finite a_w/ℓ_{dS} is to increase the strength of the effective gravitational coupling. In Fig. 4, we depict $\mathcal{A}(k)/(\ell_{\text{AdS}}/a_w)^4$ in Eq. (4.10) and approximated one in Eq. (4.37) with fixed $k = 2$ in six dimensions ($d = 5$, or $m = 1$) as a function of a_w/ℓ_{dS} . We find the approximation is good enough.

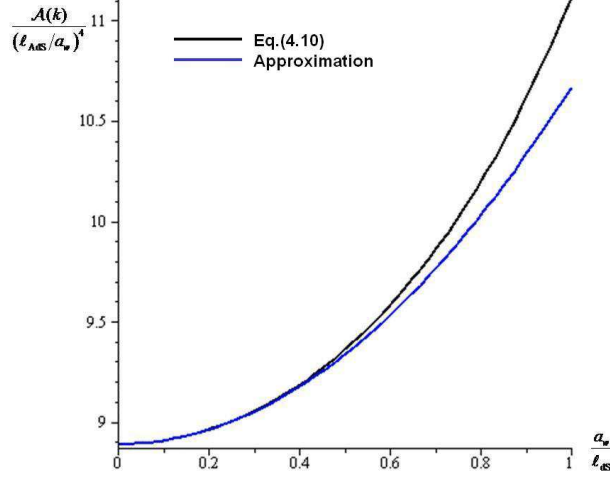


FIG. 4: The absorption probability is plotted as a function of a_w/ℓ_{dS} .

We see that the absorption probability approaches Eq. (4.20) as one approaches the Minkowski disk (to the left) and also approaches Eq. (4.35) as one moves toward the Maximal radius bubble (to the right). Thus the effect of the non-vanishing dS curvature increases the absorption probability. It is also easy to perform the perturbative analysis around the limit of AdS bubble in Minkowski spacetime $x'_0 = -1$. A similar calculation leads to the same result as given by Eqs. (4.37) and (4.38). Thus the behavior of the absorption probability near the Minkowski limit or for fairly large k can be understood from the perturbative result for the gravitational coupling constant.

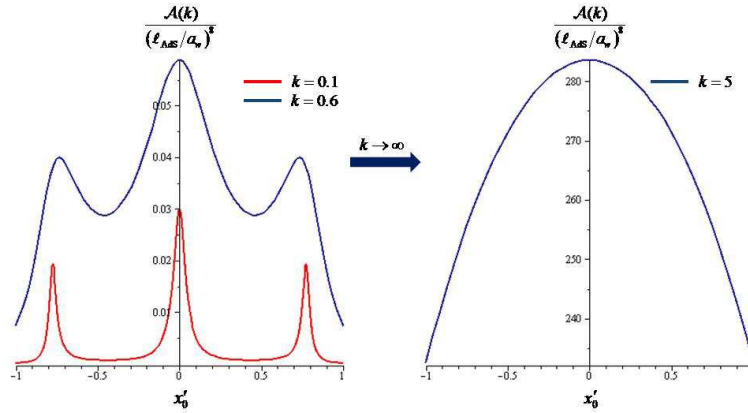


FIG. 5: The absorption probability for ten dimensions ($d = 9$ or $m = 3$) is plotted.

Although the approximation $F(a, b, 1 - ik; z) \approx 1 + \frac{ab}{1 - ik} z$ is valid for large k , it is not going to be valid for small k . In fact, for $k \leq 1$, we find qualitatively different behavior. In Figs. 5 and 6, we plot the absorption probability for

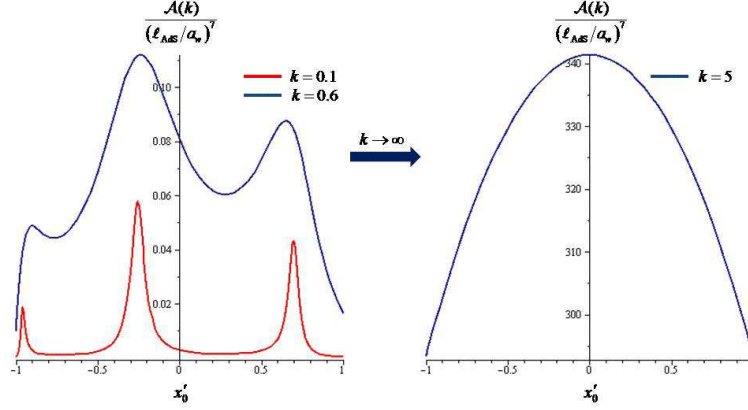


FIG. 6: The absorption probability for nine dimensions ($d = 8$, or $m = 2$) is plotted.

ten dimensions ($d = 9$ or $m = 3$) and nine dimensions ($d = 8$ or $m = 2$) at $k = 0.1$, 0.6 and $k = 5$. We clearly see resonant peaks in each figure when k is small enough. The number of the peaks is found to coincide with the number of massive graviton bound states in the limit of Minkowski bulk as discussed in Appendix C 3.

In the case of even dimensions, Fig. 5, the resonance is symmetrical with respect to $x'_0 = 0$ and it is maximum at $x'_0 = 0$ irrespective of k . On the other hand for odd dimensions, Fig. 6, the symmetry disappears and the maximum peak shifts toward a negative x'_0 . In both even and odd dimensions, the resonance gradually becomes less prominent as k increases, and merges into a single peak around $x'_0 = 0$ at $k \gg 1$.

This resonance seems to occur due to massive localized gravitons in the dS bulk. We find that the number of massive bound states of gravitons increases as the position of the domain wall moves from $x'_0 = 1$ to -1 . We confirmed this behavior by numerical calculations. More precisely, the position of each peak in Fig. 5 or Fig. 6 corresponds to the appearance of a new massive bound state as we move the domain wall from $x'_0 = 1$ to -1 .

Let us try to explain this behavior by using Fig. 5 with the help of Figs. 2 and 3. In the Minkowski disc limit $x'_0 = 1$, there is no massive bound state as discussed in Appendix C 1. The potential is flat in this limit, so the resonance does not occur. As the domain wall moves to the left from the limit, the potential well starts to appear and a bound state appear at some point. The energy of the bound state when it appears is $k^2 = 0$ and gradually it becomes $k^2 < 0$. Hence for incoming gravitons k whose energy is sufficiently small, $k \ll 1$, the resonance happens.

As the domain wall moves further towards $x'_0 = 0$, the potential well gets deep and the energy of the massive bound state decreases. Then the resonance no longer happens. However, as the potential becomes deeper a new massive bound state may appear and the resonance happens again. This makes the second peak in Fig. 5. In this case of ten dimensions, the second peak happens to be at the symmetric point $x'_0 = 0$.

Next, as the domain wall goes away further from $x'_0 = 0$ to negative x'_0 , the area of the potential well increases and the energy of the massive bound state of gravitons decreases. Then the resonance ceases to happen. However, as the area of the potential well increases as the domain wall moves toward to the Minkowski limit $x'_0 = -1$, there may appear another bound state and the resonance may occur again. This is in fact the case in ten dimensions. As the area of the potential further increases, this resonance ceases to happen again, and after the area of the potential approaches exponentially to the asymptotic value, the number of bound states no more changes. The absorption probability decreases and approaches a finite value asymptotically in the limit $x'_0 \rightarrow -1$. These features are encoded in the effective gravitational coupling \mathcal{G}_{d+1} .

E. dS/dS and dS/CFT correspondence

We have considered the dual of the dS universe with an AdS bubble, that is, the dS bulk with the boundary on which CFT resides. In this dual picture, as the AdS spacetime disappears after excision, we expect that the energy of gravitons coming from the dS bulk should be absorbed by CFT matter. We confirmed that the result for all the cases studied in the previous subsections was in agreement with the standard form of decay rate of gravitons to CFT particles in the flat wall limit. Through the calculation of absorption probability, we saw that the effect of AdS spacetime comes in only through effective degrees of freedom c_{d+1} of CFT matter. In this absorption process,

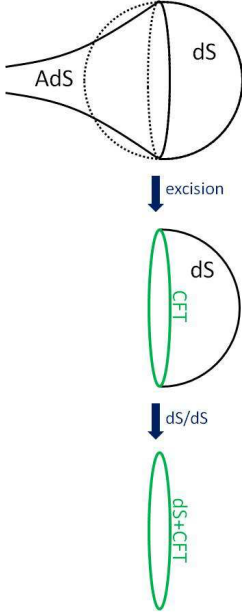


FIG. 7: The dS spacetime with the boundary CFT after excision of the AdS spacetime is also mapped onto the domain wall by the dS/dS correspondence.

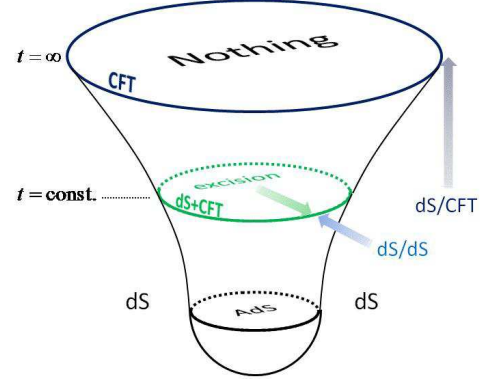


FIG. 8: A bubble evolution is depicted. After the mapping by the dS/dS correspondence, neither inside nor outside of the bubble wall exists. There exists a CFT coupled to gravity. Furthermore, we could map this onto a $(d - 1)$ -dimensional CFT in the future boundary of d -dimensional dS according to the dS/CFT correspondence.

we found that the effective gravitational coupling \mathcal{G}_{d+1} between gravitons and CFT matter is controlled by localized gravitons. As shown in Appendix C, the number of localized gravitons depends on the dimensions and the position of the domain wall. We found that for $k \gtrsim 1$ the absorption probability becomes maximal at the maximal radius bubble because the effective gravitational coupling becomes maximum there. Moreover, since the massive bound states exist, the absorption probability shows resonant peaks at low energy, $k \ll 1$. The number of resonant peaks is found to be the same as the number of the bound states in the Minkowski bulk limit. For incident gravitons with sufficiently small energy, the resonance with a bound state occurs when it appears at $k^2 = 0$. This leads to the sharp resonant decay of gravitons into CFT matter. This happens at the position where a new bound state appears.

We can further think of dual of the dS bulk with the boundary CFT, a la dS/dS correspondence [22–24], although our model is slightly different from the exact dS/dS correspondence in that there is an AdS bubble connected to the dS bulk through a domain wall. In fact, the existence of localized (massless as well as massive) gravitons in the $d + 1$ dS bulk guarantees the existence of the d -dimensional Newton constant. Thus, we can map $(d + 1)$ -dimensional gravity onto d -dimensional gravity by integrating out an extra dimension where the localized gravitons exist. Then propagating gravitons coming from the $(d + 1)$ -dimensional dS bulk turn out to be KK gravitons on the one lower dimensional domain wall. In this dS/dS correspondence picture, KK gravitons could decay into CFT matter in two ways: One is the direct decay of KK gravitons into CFT matter. The other is the indirect decay through massive gravitons, namely, KK gravitons decay first to massive gravitons, and they eventually decay into CFT matter. Therefore the effective gravitational coupling constant between KK gravitons and CFT matter is enhanced by the second process.

Also, as CFT matter lives on the domain wall, the decay probability depends on the dS radius on the wall a_w . The main difference from the usual dS/dS correspondence is that the position of the domain wall x'_0 changes the shape of potential as we see in Fig. 2 and Fig. 3. Then the mass spectrum of the localized gravitons depends on the position of the domain wall. As a consequence, the decay probability also depends on x'_0 or equivalently on ℓ_{AdS} . This is also expressed in terms of the effective gravitational coupling constant where the poles in the complex k -plane

corresponding to the graviton bound states show up. In other words, (dS with boundary)/dS correspondence can be parametrized by the position of the domain wall x'_0 . The dependence of the decay rate on x'_0 is encoded in the effective gravitational coupling constant in this dual picture.

In this (dS with boundary)/dS correspondence, we have mapped the $(d+1)$ -dimensional system into d -dimensional one. Since the geometry on the domain wall is again dS, we can utilize the dS/CFT correspondence to obtain a $(d-1)$ -dimensional CFT at the future infinity [25, 26] (see Fig. 8). The possibility of this second duality was first discussed in [13]. If we push the system into future timelike infinity, gravity will decouple from the system. Then, we can describe the whole physics in terms of the $(d-1)$ -dimensional CFT. This seems an interesting issue to study, particularly in its implications to the measure problem of the multiverse.

V. CONCLUSION

We studied the proposal that an AdS bubble in dS spacetime can be excized and replaced by a CFT at the boundary. In the original picture in which there is an AdS bubble in the dS spacetime, incident gravitons from the dS side can penetrate into the AdS bubble. We calculated the transmission probability of gravitons through the bubble wall in arbitrary dimensions. In the dual picture, the incident gravitons should be absorbed by CFT matter because there is no spacetime beyond the boundary. Therefore, the transmission probability in the original picture should be identical to the absorption probability of gravitons by CFT matter in the dual picture.

We derived a general formula for the transmission probability, that is, the absorption probability in the dual picture, in the decoupling limit of gravity in the AdS bubble, $a_w/\ell_{\text{AdS}} \gg 1$, where a_w is the wall radius and ℓ_{AdS} is the AdS curvature radius. Then we obtained the standard absorption probability in 4-dimensions in agreement with the result in [13], while we found corrections due to the curvature radius of the domain wall in higher dimensions. We argued that the fact that there were no correction in four dimensions is due to the absence of massless (ie, propagating) gravitons in three dimensions.

We found there is an important difference between the cases of even and odd spacetime dimensions. In odd dimensions, we found thermal poles obeying Fermi statistics in the amplitude of the absorption probability, while there were no thermal poles in even dimensions. We speculated that this is due to acceleration of the domain wall relative to the bulk spacetime, and there exists a universal factor that cancels thermal poles obeying Bose statistics, which happens in even spacetime dimensions, while the same factor does not cancel thermal poles obeying Fermi statistics, which is the case in odd spacetime dimensions. The apparent difference may be understood by the breakdown of the Huygens principle.

In order to reveal features of the duality, we studied various situations. Here, we summarize those briefly.

- **Minkowski disc in AdS spacetime.**

In even and odd spacetime dimensions, the absorption probability is expressed by Eqs. (4.19) and (4.21). A correction caused by the curvature radius of the domain wall, λ_{IR}^2 , comes into the formula for both cases and also a correction due to the thermal nature of the wall, $\tanh \pi k$, appears in odd spacetime dimensions. However, if we take the flat wall limit: $k = a_w k_w \rightarrow \infty$ with k_w fixed, these corrections disappear and the standard absorption probability in the Minkowski spacetime is obtained as seen in Eqs. (4.20) and (4.22). This result is consistent with Maldacena's proposal.

- **AdS bubble in Minkowski spacetime.**

The absorption probability in even and odd spacetime dimensions are given by Eqs. (4.24) and (4.28). The expression in even spacetime dimensions is the same as in the case of a Minkowski disc in AdS spacetime. On the other hand in odd spacetime dimensions, we find a curious additional correction due to the difference in the form of the hypergeometric function from that in the previous Minkowski disc case, the interpretation of which is not immediately clear. This correction is included in the effective gravitational coupling as in Eq. (4.27).

In spite of these corrections, we end up obtaining the standard absorption probability in the Minkowski spacetime again in the flat wall limit as seen in Eq. (4.29), which is consistent with Maldacena's proposal.

- **Maximal radius bubble in dS universe.**

In even spacetime dimensions, other than the same correction due to λ_{IR} as in the previous cases, we obtain the effective gravitational coupling constant, which is always greater than the original gravitational constant in the dS bulk as in Eqs. (4.33) and (4.34). The absorption probability is enhanced due to this effective gravitational coupling. We find that this enhancement is related to the bound states in the dS bulk as discussed in Appendix C. However since we have $\ell_{\text{dS}} = a_w$ by assumption in the present case, we are unable to identify if the enhanced effect is due to the dS curvature outside the bubble or due to the curvature of the domain wall. Nevertheless, considering the difference from the previous cases of Minkowski disc or Minkowski bulk, this correction can be

interpreted as the one stemming from the dS curvature outside the bubble. In spite of these corrections, we again obtain the standard absorption probability in Minkowski spacetime in the flat wall limit as in Eq. (4.35).

In odd spacetime dimensions, it is difficult to obtain a simple, analytical expression for the absorption probability. Nevertheless, the product of the absorption probabilities of two adjacent odd dimensions can be evaluated as in Eq. (4.36). The result gives the standard absorption probability in Minkowski spacetime in the flat wall limit, being consistent with Maldacena's proposal again.

- **Generic cases.**

In order to see the effect of dS curvature more explicitly, we derived an approximate formula for the general position of the domain wall in dS spacetime, valid for fairly large k , where $k = a_w k_w$ with k_w being the graviton energy on the domain wall. We also checked the formula numerically. The formula tells us that the effect of the non-vanishing dS curvature in the bulk increases the absorption probability for large k as seen in Fig. 4. In the case of small k , we find resonant peaks in the absorption probability as seen in Figs. 5 and 6. This resonance feature is encoded in the effective gravitational coupling constant where localized massive gravitons play an important role.

- **The (dS with boundary)/dS correspondence**

The interpretation of this correspondence is natural in that bulk gravitons are interpreted as KK gravitons in lower dimensions. More precisely, our absorption probability formula gives the decay probability of KK gravitons through two channels, one directly decaying to CFT matter and the other indirectly via localized massive graviton modes. Here, we mention that the wall fluctuation mode may have some relevance to localized massive modes, and discussed in Appendix D.

In conclusion, the calculation of the absorption rate of gravitons shows that Maldacena's excision proposal is physically reasonable. However, we also find non-trivial correction factors due to finiteness of the wall radius and/or the dS curvature. We discussed the origin and meaning of these corrections. Apparently it is interesting to understand more clearly the role of the localized modes of gravitons in this duality. It may be also interesting to study the relation of our approach to other approaches [15, 27].

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Appendix A: Formulas related to associated Legendre functions

The definition of the associated Legendre function $P_\nu^\mu(u)$ we adopt in this paper is

$$P_\nu^\mu(x) = \frac{1}{\Gamma(1-\mu)} \left(\frac{x+1}{x-1} \right)^{\mu/2} F \left(-\nu, \nu+1, 1-\mu; \frac{1-x}{2} \right) \quad \text{for } u > 1, \quad (\text{A1})$$

and

$$P_\nu^\mu(x) = \frac{e^{i\pi\mu}}{\Gamma(1-\mu)} \left(\frac{1+x}{1-x} \right)^{\mu/2} F \left(-\nu, \nu+1, 1-\mu; \frac{1-x}{2} \right) \quad \text{for } u < 1, \quad (\text{A2})$$

where $F(a, b, c; u)$ is the hypergeometric function defined by

$$F(a, b, c; u) = \frac{\Gamma(c)}{\Gamma(a)\Gamma(b)} \sum_{n=0}^{\infty} \frac{\Gamma(a+n)\Gamma(b+n)}{\Gamma(c+n)} \frac{u^n}{n!}. \quad (\text{A3})$$

This series is known to be convergent for $|u| < 1$ on the complex u -plane.

By definition $F(a, b, c; 0) = 1$ for any a, b and c . Hence the behavior of $P_\nu^\mu(x)$ in the limit $x \rightarrow 1$ is completely governed by the factor $((x+1)/(x-1))^{\mu/2}$ or $((1+x)/(1-x))^{\mu/2}$ in front of the hypergeometric function. In general, $F(a, b, c; u)$ is singular at $u \rightarrow 1$. However, it may have a regular limit if both $\Re c > 0$ and $\Re c - a - b > 0$ are satisfied. In this case,

$$F(a, b, c; 1) = \frac{\Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)}, \quad (\text{A4})$$

and the behavior of $P_\nu^\mu(x)$ in the limit $x \rightarrow -1$ is governed by the factor $((x+1)/(x-1))^{\mu/2}$ or $((1+x)/(1-x))^{\mu/2}$.

The hypergeometric function can be analytically continued to the region $|u| > 1$ of the complex u -plane. However in general there is a branch cut emanating from $u = 1$ toward infinity. Usual convention is to place a branch cut on the real axis; $u \in [1, \infty]$. The analytically continued function can be again expressed in terms of the hypergeometric functions. There are various analytic continuation formulas, but one of the most useful formulas is

$$\begin{aligned} F(a, b, c; u) = & \frac{\Gamma(b-a)\Gamma(c)}{\Gamma(b)\Gamma(c-a)}(-u)^{-a}F(a, a-c+1, a-b+1; 1/u) \\ & + \frac{\Gamma(a-b)\Gamma(c)}{\Gamma(a)\Gamma(c-b)}(-u)^{-b}F(b, b-c+1, b-a+1; 1/u) \quad (|u| > 1, u \neq \text{real}). \end{aligned} \quad (\text{A5})$$

As clear from the above definition, the dependence of $P_\nu^\mu(x)$ on the lower index ν is totally encoded in the hypergeometric function F . Since the hypergeometric function has the symmetry between the first two arguments, $F(a, b, c; u) = F(b, a, c; u)$, it follows that $P_\nu^\mu(x)$ has the symmetry,

$$P_\nu^\mu(x) = P_{-\nu-1}^\mu(x). \quad (\text{A6})$$

Another useful property is that the definition of the hypergeometric function (A3) implies $F(0, b, c; u) = F(b, 0, c; u) = 1$ independent of u , and if either of a or b is a negative integer, the series in (A3) terminates at a finite order, that is, it simplifies to a polynomial of a finite order. Therefore, apart from this finite polynomial factor, the behavior of $P_\nu^\mu(x)$ is again governed by the factor $((x+1)/(x-1))^{\mu/2}$ or $((1+x)/(1-x))^{\mu/2}$.

Finally, we recapitulate the derivative formula,

$$\begin{aligned} (1-x^2)\frac{dP_\nu^\mu(x)}{dx} = & (\nu+1)xP_\nu^\mu(x) - (\nu-\mu+1)P_{\nu+1}^\mu(x) \\ = & (\nu+\mu)P_{\nu-1}^\mu(x) - \nu xP_\nu^\mu(x), \end{aligned} \quad (\text{A7})$$

and the recurrence relation,

$$(\nu+1-\mu)P_{\nu+1}^\mu(x) - (2\nu+1)xP_\nu^\mu(x) + (\nu+\mu)P_{\nu-1}^\mu(x) = 0. \quad (\text{A8})$$

Appendix B: Exact transmission probability formula in four dimensions

Here we derive an exact formula for the transmission probability in four dimensions without taking the decoupling limit.

Using the function $F_{\nu,k}(x)$ defined in Eq. (4.5),

$$F_{\nu,k}(x) = F\left(-\nu, \nu+1, 1-ik; \frac{1-x}{2}\right), \quad (\text{B1})$$

the original general formula for \mathcal{M} , Eq. (3.17), can be cast in the form,

$$\mathcal{M}^{-1} = -e^{-\pi k} \frac{\nu+ik}{\Gamma^2(1-ik)} \left(\frac{x_0+1}{x_0-1}\right)^{ik/2} \left(\frac{1+x'_0}{1-x'_0}\right)^{ik/2} \left(F_{\nu,k}(x_0)F_{\nu-1,k}(x'_0) + F_{\nu-1,k}(x_0)F_{\nu,k}(x'_0)\right), \quad (\text{B2})$$

where we recall that

$$x_0 = \coth z_0 = \sqrt{\left(\frac{a_w}{\ell_{\text{AdS}}}\right)^2 + 1}, \quad x'_0 = \tanh z'_0 = \pm \sqrt{1 - \left(\frac{a_w}{\ell_{\text{ds}}}\right)^2}. \quad (\text{B3})$$

Now we focus on the case of four dimensions, $d = 3$. In this case we have $\nu = (d - 1)/2 = 1$, and hence $F_{\nu-1,k}(x)$ is trivial, $F_{0,k}(x) = 1$, and $F_{1,k}(x)$ simplifies to

$$F_{1,k}(x) = F\left(-1, 2, 1 - ik; \frac{1-x}{2}\right) = 1 - \frac{2}{1-ik} \frac{1-x}{2}. \quad (\text{B4})$$

Thus we find

$$\begin{aligned} \mathcal{M}_{d=3}^{-1} &= -e^{-\pi k} \frac{1+ik}{\Gamma^2(1-ik)} \left(\frac{x_0+1}{x_0-1}\right)^{ik/2} \left(\frac{1+x'_0}{1-x'_0}\right)^{ik/2} (F_{1,k}(x_0) + F_{1,k}(x'_0)) \\ &= -e^{-\pi k} \frac{1+ik}{\Gamma^2(1-ik)} \left(\frac{x_0+1}{x_0-1}\right)^{ik/2} \left(\frac{1+x'_0}{1-x'_0}\right)^{ik/2} \frac{x_0+x'_0-2ik}{1-ik}. \end{aligned} \quad (\text{B5})$$

Therefore

$$|\mathcal{M}_{d=3}|^2 = \frac{\pi^2 e^{2\pi k}}{\sinh^2 \pi k} \frac{k^2}{(x_0+x'_0)^2 + 4k^2}. \quad (\text{B6})$$

Inserting this into the transmission probability formula (3.16), we obtain

$$\mathcal{T}_{d=3} = \frac{4k^2}{(x_0+x'_0)^2 + 4k^2}. \quad (\text{B7})$$

Note that we have not used any approximation, hence this is an exact result, valid for any k for any values of x_0 and x'_0 . If we recall the formula for the tension of the wall T , Eq. (2.5), we have

$$x_0 + x'_0 = \frac{a_w}{\ell_{\text{AdS}}} \frac{T}{T_{\text{cr}}} = \frac{\kappa^2}{2} a_w T. \quad (\text{B8})$$

Therefore the above may be expressed in the form,

$$\mathcal{T}_{d=3} = \frac{k^2}{A^2 + k^2}; \quad A \equiv \frac{\kappa^2}{4} a_w T. \quad (\text{B9})$$

We see this completely agrees with Eq. (29) of Ref. [13] in the Minkowski bulk limit, $x'_0 \rightarrow -1$.

Appendix C: Bound states

Let us consider graviton bound states of Eq. (3.5) in the $(d+1)$ -dimensional spacetime. In contrast to the propagating solution given by Eqs.(3.10) and (3.11) whose eigenvalue satisfies $M^2 > \nu^2$, a bound state has an eigenvalue $M^2 \leq \nu^2$ (or $k^2 \leq 0$), and the boundary condition is that it should vanish at the $z \rightarrow \pm\infty$. Thus, we have

$$\phi_{\text{AdS}} = A P_\nu^{-\mu}(\coth(z_0 - z)), \quad (\text{C1})$$

$$\phi_{\text{dS}} = B P_\nu^{-\mu}(\tanh(z + z'_0)), \quad (\text{C2})$$

where

$$\mu = \sqrt{\nu^2 - M^2}, \quad \nu \equiv \frac{d-1}{2}. \quad (\text{C3})$$

The two junction conditions given as by Eqs. (3.13) and (3.14) at $z = 0$ are now in the form,

$$A P_\nu^{-\mu}(x_0) = B P_\nu^{-\mu}(x'_0), \quad (\text{C4})$$

$$\begin{aligned} A [(\nu + 1 + \mu) P_{\nu+1}^{-\mu}(x_0) - (\nu + 1)x_0 P_\nu^{-\mu}(x_0) - \nu(x_0 + x'_0) P_\nu^{-\mu}(x_0)] \\ = B [-(\nu + 1 + \mu) P_{\nu+1}^{-\mu}(x'_0) + (\nu + 1)x'_0 P_\nu^{-\mu}(x'_0)], \end{aligned} \quad (\text{C5})$$

where $x_0 = \coth z_0$ and $x'_0 = \tanh z'_0$ as defined in Eq. (3.15). In order to have a non-trivial solution for A and B , we require the condition,

$$(\nu + 1 + \mu) [P_\nu^{-\mu}(x_0) P_{\nu+1}^{-\mu}(x'_0) + P_{\nu+1}^{-\mu}(x_0) P_\nu^{-\mu}(x'_0)] = d(x_0 + x'_0) P_\nu^{-\mu}(x_0) P_\nu^{-\mu}(x'_0). \quad (\text{C6})$$

Note that if we compare the above with the propagating case, we see that a bound state corresponds to a pure imaginary pole at $k = i\mu$ ($\mu \geq 0$) in the transmission coefficient \mathbf{T} on the complex k -plane.

For $x_0 \gg 1$, which is the decoupling limit of gravity as in Eq. (4.1), the above condition reduces to

$$(\nu - \mu) P_{\nu-1}^{-\mu}(x'_0) = 0, \quad (\text{C7})$$

where we have used the same recurrence relation given below Eq. (3.17).

There is at least one bound state solution given by $\mu = \nu$, namely the zero mode solution, $M^2 = 0$, independent of the wall position x'_0 . In fact the zero mode solution exists for any value of x_0 as well, and it is simply given in terms of the scale factor as $\phi_0(z) \propto a^\nu(z)$.

In addition to the zero mode, there may be a massive bound state $0 < M^2 \leq \nu^2$ if we have

$$P_{\nu-1}^{-\mu}(x'_0) = \frac{e^{-\mu\pi i}}{\Gamma(1+\mu)} \left(\frac{1-x'_0}{1+x'_0} \right)^{\mu/2} F\left(-\nu+1, \nu, 1+\mu; \frac{1-x'_0}{2}\right) = 0, \quad (\text{C8})$$

for a given value of x'_0 .

In the case of four spacetime dimensions, $d = 3$ ($\nu = 1$), the hypergeometric function in the above equation becomes trivial $F(0, 1, c; x) = 1$, hence it is satisfied only when $x'_0 = 1$. In this case, however, $F(-\nu+1, \nu, 1+\mu; 0) = 1$ for any value of ν , hence the dependence of $P_{\nu-1}^{-\mu}$ on ν disappears and we have $P_{\nu-1}^{-\mu} = P_{\nu}^{-\mu} = 0$. Then from Eq. (C4), the solution becomes trivial $A = 0$. Thus, there is no solution other than the zero mode irrespective of x'_0 for $d = 3$. However, since there is no dynamical massless graviton in three spacetime dimensions, this zero mode is a mathematical artifact. In fact, if we go back to the original tensor equation in d -dimensions, Eq. (3.4), one finds there is no dynamical solution for Y_{ij} when $d = 3$.

We now try to find massive bound state solutions in higher dimensions $d > 3$ ($\nu > 1$) in the three limiting cases of $x'_0 = \pm 1$ and 0.

1. Minkowski disc limit

For the Minkowski disc limit $x'_0 = 1$, we have $P_{\nu-1}^{-\mu} = 0$ in Eq. (C8) irrespective of the value of μ . However, in complete analogy with the $d = 3$ case discussed above, since the ν dependence of $P_{\nu-1}^{-\mu}$ disappears and we have $P_{\nu-1}^{-\mu} = P_{\nu}^{-\mu} = 0$. Then Eq. (C4) gives a trivial solution $A = 0$. Therefore there is no bound state solution in the Minkowski disc limit except for the zero mode,

$$M^2 = 0. \quad (\text{C9})$$

2. Maximal radius bubbles

If the domain wall moves away from $x'_0 = 1$, massive bound state solutions may appear for $d > 3$. Since an analytical study is difficult for general values of x'_0 , we consider the case of a maximal radius bubble $x'_0 = 0$.

In this case, we can solve Eq. (C8) analytically if we recall the formula (4.31). Setting $a = \nu - 1$ and $b = 1 + \mu$, we find that the factor $\Gamma((b-a)/2)$ in the denominator diverges at $\mu - \nu + 2 = -2n'$ ($n' = 0, 1, 2, \dots$). Combining this with the relation $\mu = \sqrt{\nu^2 - M^2}$, we obtain the mass spectrum as

$$M^2 = \nu^2 - (\nu - 2n)^2; \quad n = 1, 2, \dots, \left\lfloor \frac{\nu}{2} \right\rfloor, \quad (\text{C10})$$

where $[Q]$ is the maximum integer not exceeding Q .

The above condition determines the number of massive bound states of gravitons. In order for the n -th massive bound state to exist, we need $\nu \geq 2n$ or $d \geq 4n + 1$. Therefore, the minimum spacetime dimensions to have a massive mode is six ($d = 5$). As discussed in Section IV D, there is one massive bound state ($n = 1$) in nine dimensions ($d = 8$ or $m = 2$ where $d = 2m + 4$), and two bound states ($n = 1, 2$) in ten dimensions ($d = 9$ or $m = 3$ where $d = 2m + 3$).

3. Minkowski bulk limit

For the Minkowski bulk limit $x'_0 \rightarrow -1$, we use the formula,

$$P_{\nu-1}^{-\mu}(x) = e^{-\mu\pi i} \left(\frac{1-x}{1+x} \right)^{\mu/2} \left[\frac{\Gamma(-\mu)}{\Gamma(-\nu+1)\Gamma(\nu)} \left(\frac{1+x}{2} \right)^{\mu} F \left(\mu+\nu, \mu-\nu+1, 1+\mu; \frac{1+x}{2} \right) + \frac{\Gamma(\mu)}{\Gamma(\mu-\nu+1)\Gamma(\mu+\nu)} F \left(-\nu+1, \nu, 1-\mu; \frac{1+x}{2} \right) \right]. \quad (\text{C11})$$

We see that $\Gamma(\mu-\nu+1)$ in the second term in the square brackets diverge at $\mu = \nu - 1 - n'$ with $n' = 0, 1, 2, \dots$. Then, we obtain the mass spectrum,

$$M^2 = \nu^2 - (\nu - n)^2; \quad n = 1, 2, \dots, [\nu]. \quad (\text{C12})$$

The case when $n = \nu$ for an integer ν needs some care. Since $\mu = 0$ in this case, the above formula is inapplicable. However, if we remember that the case $\mu = 0$ with integer ν is simply a Legendre polynomial $P_n(x)$, we immediately see that $\mu = 0$ is not a bound state solution because $P_n(-1) = (-1)^n \neq 0$.

Hence the number of the bound states is $[\nu - 1/2]$ instead of $[\nu]$. Namely, for the n -th bound state to exist we need $\nu \geq (n+1)$ or $d \geq 2n+2$. Thus, the minimum dimensions to have a massive mode in is five ($d = 4$). There are three bound states ($n = 1, 2, 3$) both in nine dimensions ($d = 8$ or $m = 2$ where $d = 2m+4$) and in ten dimensions ($d = 9$ or $m = 3$ where $d = 2m+3$). We note that this result coincides with that of the pure dS case.

In conclusion, we found that there exists a massless bound state independent of the position of the domain wall x'_0 in all dimensions higher than four ($d+1 \geq 5$), while there may exist massive bound states and the number of them varies depending on the position of the domain wall x'_0 and the dimension d . The number of bound states increases as x'_0 varies from 1 to -1 and also as d increases. We have confirmed this with numerical calculations. As discussed in Section IV D, we found numerically that there appears a resonance peak in the absorption probability every time a new bound state appears as x'_0 varies from 1 to -1 for a given d .

Appendix D: wall fluctuation mode

First recall that our $(d+1)$ -dimensional metric is given by

$$ds^2 = a^2(z) [dz^2 + ds_{\text{dS}}^2], \quad (\text{D1})$$

where

$$ds_{\text{dS}}^2 = g_{ij} dx^i dx^j = -d\tau^2 + \cosh^2 \tau d\Omega_{(d-1)}^2, \quad (\text{D2})$$

is a dS spacetime with the Ricci tensor given by

$$R_{ij} = (d-1)g_{ij}. \quad (\text{D3})$$

On this background we place a domain wall at $z = z_w$, and consider fluctuations of the position of the wall, $z = z_w + \psi(\tau, x^i)$. Thus $\psi(x^i)$ will behave as a scalar field in d dimensions. Let the mass of ψ be M_s . The field equation is

$$[\square - M_s^2] \psi = 0. \quad (\text{D4})$$

Now consider a traceless tensor Q_{ij} constructed from ψ ,

$$Q_{ij} = \left[\nabla_i \nabla_j - \frac{g_{ij}}{d} \square \right] \psi. \quad (\text{D5})$$

Then

$$\begin{aligned} \nabla^i Q_{ij} &= \nabla^i \left[\nabla_j \nabla_i - \frac{g_{ij}}{d} \square \right] \psi = \left[(\nabla_i \nabla_j - \nabla_j \nabla_i) \nabla^i + \nabla_j \square - \frac{1}{d} \nabla_j \square \right] \psi \\ &= \left[R_{ij} \nabla^i + M_s^2 \frac{d-1}{d} \nabla_j \right] \psi = (d-1) \left[1 + \frac{M_s^2}{d} \right] \psi. \end{aligned} \quad (\text{D6})$$

Hence Q_{ij} becomes transverse-traceless if $M_s^2 = -d$. This tachyonic mode is known to be the wall fluctuation mode (at least in the limit of Minkowski bulk).

If we calculate $\square Q_{ij}$ using Eq. (D3), we find

$$\square Q_{ij} = (M_s^2 + 2d)Q_{ij}. \quad (\text{D7})$$

Comparing this with Eq. (3.4), we see that the scalar mass-squared M_s^2 would correspond to the tensor mass-squared M^2 as

$$M^2 + 2 = M_s^2 + 2d \quad \leftrightarrow \quad M^2 = M_s^2 + 2d - 2. \quad (\text{D8})$$

For the tachyonic mass $M_s^2 = -d$, this gives

$$M^2 = -d + 2d - 2 = d - 2. \quad (\text{D9})$$

Thus the wall fluctuation mode would correspond to the mode $M^2 = d - 2$. In terms of μ with $\mu^2 = \nu^2 - M^2$, this corresponds to

$$\mu = \sqrt{\nu^2 - M^2} = \sqrt{\nu^2 - (d - 2)} = \frac{d - 3}{2} = \nu - 1. \quad (\text{D10})$$

The first massive localized mode in the Minkowski bulk limit obtained in Appendix C 3 is nothing but this wall fluctuation mode. Curiously, this mode disappears in the other two cases.

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